

Numerical Methods for Partial Differential Equations

Homework 3

Problem 1

We consider the Cauchy problem

$$u_t + \left(\frac{1}{2}u^2\right)_x = 0$$

in $\mathbb{R} \times \mathbb{R}_0^+$ with initial data

$$u_0(x) = \begin{cases} 2, & x \leq 0, \\ 2 - x, & 0 < x < 1, \\ 1, & x \geq 1. \end{cases}$$

Determine and plot the solution for $t = 0.5, 1, 1.5, 2$. (4 P)

Problem 2

Compute the solution $\mathbf{u}(x, 1/2)$ of the Riemann problem

$$\partial_t \mathbf{u}(x, t) + \mathbf{A} \partial_x \mathbf{u}(x, t) = \mathbf{0}$$

in $\mathbb{R} \times \mathbb{R}_0^+$ with

$$\mathbf{A} = \begin{pmatrix} 2 & 0 \\ -1/2 & 3 \end{pmatrix}.$$

The initial data is given as

$$\mathbf{u}(x, 0) = \begin{pmatrix} 4 \\ 1 \end{pmatrix},$$

for $x < 0$ and

$$\mathbf{u}(x, 0) = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

for $x \geq 0$. (4 P)

Due on Friday, April 27, 2012.