University of Kassel, Institute of Mathematics **Summer Term 2012**
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Numerical Methods for Partial Differential Equations

Homework 4

Problem 1
We consider the one-dimensional advection-diffusion equation

$$u_t + \beta u_x = \alpha u_{xx}$$

on $(0, 10) \times \mathbb{R}_0^+$ with boundary conditions $u(0, t) = u(10, t) = 0$ and initial data

$$u(x, 0) = \begin{cases} 1, & 4.9 \leq x \leq 5.1 \\ 0, & \text{elsewhere.} \end{cases}$$

Write a code for the computation of an approximate solution of the above equation using finite differences in space and explicit Euler method in time. Compute the solution at time $t = 1$ with $\Delta x = 0.1$ and $\Delta t = 0.01$ in the following three cases:

a) Advection: $\alpha = 0$ and $\beta = 2$

b) Diffusion: $\alpha = 0.1$ and $\beta = 0$

c) Advection-Diffusion: $\alpha = 0.1$ and $\beta = 2$

Do this for each of the three discretizations of the advection term mentioned in class (forward, central and backward). What do you observe? (4 P)

Problem 2
Determine the dependence of the time step size on the mesh width and the parameters α and β experimentally.

- Considering the case of pure advection, compute the maximum time step size $\Delta t_{\max}(N)$ with $\beta > 0$ fixed and $\Delta x = \frac{10}{N+1}$, such that your code produces reasonable results. Furthermore, compute $\Delta t_{\max}(\beta)$ with Δx fixed and $\beta > 0$. Here we consider a solution to be reasonable if it is bounded between 0 and 1.

- Do the same in the case of pure diffusion, but this time for α instead of β. Here we consider a solution to be acceptable if it is smooth.

- With respect to advection-diffusion, only compute the dependence of the time step for α as well as β fixed.

In all these numerical experiments you can restrict to a backward difference discretization of the advection term. Visualize your results and prepare a report which can be presented during the exercise on Monday. (4 P)

Due on Friday, Mai 4, 2012.