University of Kassel, Institute of Mathematics **Summer Term 2012**
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Numerical Methods for Partial Differential Equations

Homework 5

Problem 1
Show that the upwind discretization (backward differences) of the linear advection equation

$$\partial_t u + a\partial_x u = 0, \quad a > 0$$

can be interpreted as an approximation of the advection-diffusion equation

$$\partial_t u + a\partial_x u = \epsilon\partial_{xx} u, \quad a > 0$$

with central differences. Here ϵ needs to be defined properly. (4 P)

Problem 2
The Lax-Wendroff scheme for the numerical approximation of the lineare advection equation is given by

$$u_j^{n+1} = u_j^n - \frac{\lambda a}{2}\left(u_{j+1}^n - u_{j-1}^n\right) + \frac{\lambda^2 a^2}{2}\left(u_{j+1}^n - 2u_j^n + u_{j-1}^n\right)$$

with $\lambda = \frac{\Delta t}{\Delta x}$. Determine the method's order of consistency in space and time. (4 P)

Due on Friday, May 11, 2012.