University of Kassel, Institute of Mathematics **Summer Term 2012**
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Numerical Methods for Partial Differential Equations

Homework 6

Problem 1
Show that the Lax-Wendroff method

$$u_j^{n+1} = u_j^n - \frac{\lambda a}{2} \underbrace{\left(u_{j+1}^n - u_{j-1}^n\right)}_{=F_1^{LW}} + \frac{\lambda^2 a^2}{2} \underbrace{\left(u_{j+1}^n - 2u_j^n + u_{j-1}^n\right)}_{=F_2^{LW}}$$

with $a > 0$ and $\lambda = \frac{\Delta t}{\Delta x}$ is stable if the CFL condition $a\lambda < 1$ is satisfied. (4 P)

Problem 2
We consider the linear advection equation

$$\partial_t u + a\partial_x u = 0, \quad x \in [0, 25]$$

with inital data

$$u_0(x) = \exp(-20(x-2)^2) + \exp(-(x-5)^2)$$

and $u(0,t) = 0$, $a = 1$. Compute the approximate solution at $t = 17$ with the upwind scheme, the Lax-Wendroff method, as well as the Beam-Warming scheme

$$u_j^{n+1} = u_j^n - \frac{\lambda a}{2} \underbrace{\left(3u_j^n - 4u_{j-1}^n + u_{j-2}^n\right)}_{=F_1^{BW}} + \frac{\lambda^2 a^2}{2} \underbrace{\left(u_j^n - 2u_{j-1}^n + u_{j-2}^n\right)}_{=F_2^{BW}},$$

and the method of Fromm

$$u_j^{n+1} = u_j^n - \frac{\lambda a}{2} \frac{\left(F_1^{LW} + F_1^{BW}\right)}{2} + \frac{\lambda^2 a^2}{2} \frac{\left(F_2^{LW} + F_2^{BW}\right)}{2}.$$

Use the mesh width $\Delta x = 0.05$ and time step size $\Delta t = 0.8\Delta x$. Plot the approximation together with the exact solution in the interval [15,25]. What do you observe? (4 P)

Due on Friday, May 18, 2012.