University of Kassel, Institute of Mathematics **Summer Term 2012**
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Numerical Methods for Partial Differential Equations

Homework 6

Problem 1
Write a program for the solution of the one dimensional Euler equations

$$\partial_t \begin{pmatrix} \varrho \\ \varrho v \\ \varrho E \end{pmatrix} + \partial_x \begin{pmatrix} \varrho v \\ \varrho v^2 + p \\ (\varrho E + p)v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

using the numerical flux functions of Lax-Friedrichs, Steger-Warming and van Leer. The time step size shall be defined as

$$\Delta t = \frac{\mathrm{CFL}\Delta x}{\max_{i=1,\dots,N}(|v_i| + c_i)}.$$

Compute the solution of the Riemann problem

$$\mathbf{u_0}(\mathbf{x}) = \begin{cases} \mathbf{u}_L, & 0 \le x \le 0.3, \\ \mathbf{u}_R, & 0.3 < x \le 1, \end{cases}$$

and

$$\mathbf{u}_L = \begin{pmatrix} \varrho_L \\ v_L \\ p_L \end{pmatrix} = \begin{pmatrix} 1 \\ 0.75 \\ 1 \end{pmatrix}, \quad \mathbf{u}_R = \begin{pmatrix} \varrho_R \\ v_R \\ p_R \end{pmatrix} = \begin{pmatrix} 0.125 \\ 0 \\ 0.1 \end{pmatrix},$$

at time $t = 0.2$. Use $\Delta x = 1/1000$ and CFL=0.9. Plot the primitive variables ρ, v and p and compare your results to the reference solution from the web page. (8 P)

Due on Friday, May 25, 2012.